CHAPTER 14 -- GAUSS'S LAW

14.1) This problem is tricky. An electric field line that flows into, then out of the cap (see Figure I) produces a negative flux when entering and an equal positive flux when exiting. Its net flux equals ZERO. That means that the only electric field lines that will produce a net flux are those that enter through the body of the cap while exiting through the cap's hole (they produced negative flux while passing in but produce no additional flux as they exit via the hole). As the hole has an area A_o, the flux through the hole, hence through the cap, will equal EA_o.

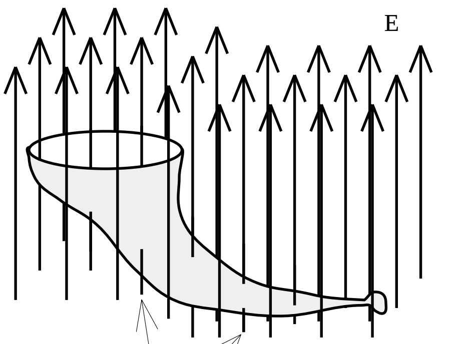


FIGURE I

14.2) The volume charge density for this charge configuration will be:

$$\rho = \frac{Q}{\left(\frac{4}{3}\pi R^3\right)} = \frac{3Q}{4\pi R^3}.$$

Additionally, a differential shell of charge of radius b and differential thickness db located inside the Gaussian surface will have a differential volume dV equal to:

$$dV = (\text{surface area})(\text{thickness}) = (4\pi b^2)db.$$

a.) For a spherical Gaussian surface inside the sphere (see Figure II to the right), we can write:

$$\int_S \mathbf{E} \bullet d\mathbf{S} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Rightarrow \quad \int_S EdS = \frac{\int_{b=0}^{r} \rho dV}{\varepsilon_o}.$$

cross-section of spherical shell

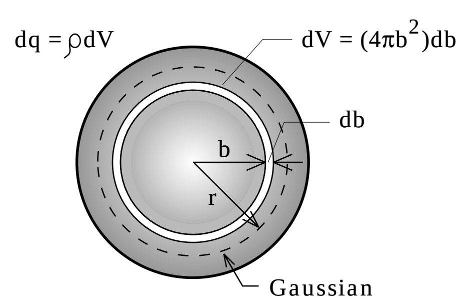


FIGURE II

$$\Rightarrow \quad E\int_S dS = \frac{\int_{b=0}^{r}\left[\frac{3Q}{4\pi R^3}\right]\left[\left(4\pi b^2\right)db\right]}{\varepsilon_o}$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\left[\frac{3Q}{R^3}\right]\int_{b=0}^{r}(b^2)db}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\left[\frac{3Q}{R^3}\right]\left[\frac{b^3}{3}\right]_{b=0}^{r}}{4\pi r^2\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{Qr}{4\pi\varepsilon_o R^3}.$$

b.) For the field outside the sphere: A Gaussian surface outside the sphere will have Q's worth of charge within it. For this situation, Gauss's Law in truncated form becomes:

$$\int_S \mathbf{E} \bullet d\mathbf{S} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{Q}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{Q}{4\pi\varepsilon_o r^2}.$$

In other words, the sphere will appear like a point charge for $r > R$.

14.3) The situation is sketched to the right.

a and b.) Assume for the moment that the +Q's worth of charge was NOT placed at the center of the system. In that case, the +2Q's worth of charge placed on the inside surface of the conductor would migrate to the outside surface of the conductor due to electrostatic repulsion.

Assume now that the +2Q's worth of charge was not placed on the inside surface of the conductor, but that the +Q's worth of charge was placed at the center of the system. In that case, -Q's worth of charge would migrate from the outer surface of the conductor to the inner surface of the conductor, leaving +Q's worth of charge on the outside surface. How do we know this? Because the electric field inside the conductor must be zero for static charge situations, and the only way that can happen is if the net charge internal to a Gaussian surface inside the conducting region is zero. That will only happen if the charge at the center and the charge induced onto the inside surface of the conductor are equal and opposite.

The two situations together yield the following: The +Q's worth of charge levitating at the center will induce -Q's worth of charge onto the inner surface of the conductor, leaving +Q's worth of charge on the outer surface of the conductor. That will be joined by the +2Q's worth of charge that was initially placed on the conductor's inside surface, but that subsequently migrated to the outer surface due to electrostatic repulsion.

<ins>Bottom line</ins>: There will be +Q's worth of charge at the center, -Q's worth of charge on the inside surface of the conductor, and +3Q's worth of charge on the outside surface of the conductor.

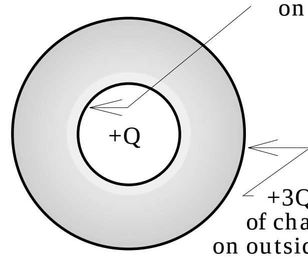


FIGURE III

c.) The charge enclosed inside a spherical Gaussian surface whose radius is $r < R_1$ is +Q. As such, we can write:

$$\int_S \mathbf{E} \bullet d\mathbf{S} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{Q}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{Q}{4\pi\varepsilon_o r^2}.$$

d.) The region is a conductor. As such, the electric field in the region is zero. Proof? The charge inside a Gaussian surface within the region will be -Q + Q = 0.

e.) For $r > R_2$ the charge enclosed within a Gaussian surface will be +3Q. As such, we can write:

$$\int_S \mathbf{E} \bullet d\mathbf{S} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{3Q}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{3Q}{4\pi\varepsilon_o r^2}.$$

f.) The surface charge density on a sphere of radius R_2 having a total of +3Q's worth of charge on it is:

$$\sigma = \frac{(3Q)}{4\pi R_2^{\,2}}.$$

14.4) The system is shown in Figure IV.

a.) The units of k must be inverse meters as the unit for r is meters and the exponent (i.e., kr) must be unitless. The units of C must be coulombs per meter if ρ is to be a volume charge density.

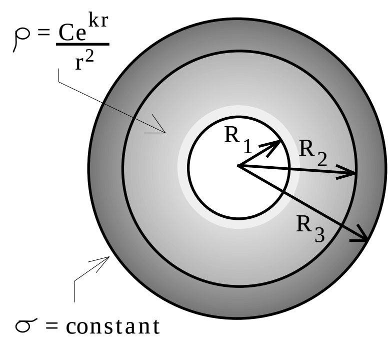


FIGURE IV

b.) To determine the Electric Field versus Position graph, we must determine the electric field function for each of the four regions in which distinct fields exist. Using Gauss's Law in conjunction with spherical Gaussian surfaces for each region, we get:

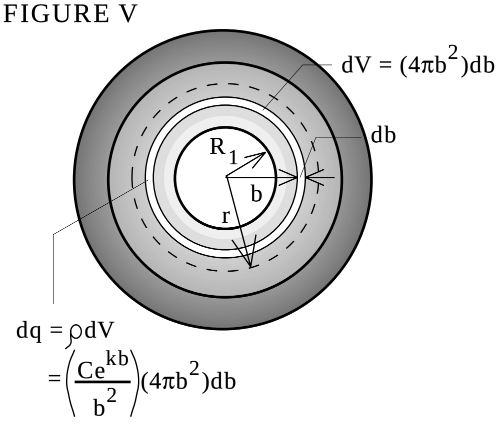
--For $r < R_1$: The electric field in this region is ZERO as there is no charge enclosed within it.

--For $R_1 < r < R_2$: Assuming the spherical Gaussian surface in this region has a radius equal to r, we need to determine the amount of charge residing inside that surface.

The volume charge density b units from the sphere's center is:

$$\rho = \frac{C}{b^2}e^{kb}.$$

FIGURE V



If we can determine the differential charge inside a spherical shell of radius b and thickness db, then we can integrate that between the inside radius of the shell (i.e., R_1) and the Gaussian radius r. Figure V highlights the setup.

$$\int_S \mathbf{E} \bullet d\mathbf{S} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Rightarrow \quad \int_S E dS = \frac{\int_{b=R_1}^{r} \rho dV}{\varepsilon_o}$$

$$\Rightarrow \quad E\int_S dS = \frac{\int_{b=R_1}^{r} \left[\frac{C}{b^2} e^{kb}\right]\left[\left(4\pi b^2\right) db\right]}{\varepsilon_o}$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{4\pi C\int_{b=R_1}^{r} (e^{kb}) db}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{4\pi C\left(\frac{1}{k}\right)\left[e^{kb}\right]_{b=R_1}^{r}}{4\pi r^2 \varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\left(\frac{C}{k}\right)\left[e^{kr} - e^{kR_1}\right]}{r^2 \varepsilon_o}.$$

--For $R_2 < r < R_3$: Inside the conductor, the electric field will equal ZERO.

--For $r > R_3$: The Gaussian surface encloses all the charge within the configuration including that on the outer surface of the conductor. In truncated form, Gauss's Law for this yields:

$$\int_S \mathbf{E} \bullet d\mathbf{S} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Rightarrow \quad E\int_S dS = \frac{\int_{b=R_1}^{R_2} \left[\frac{C}{b^2} e^{kb}\right]\left[\left(4\pi b^2\right) db\right] + \sigma(4\pi {R_3}^2)}{\varepsilon_o}$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{4\pi C\int_{b=R_1}^{R_2} (e^{kb}) db + \sigma(4\pi {R_3}^2)}{\varepsilon_o}$$

$$\Rightarrow \qquad E = \frac{4\pi C\left(\frac{1}{k}\right)\left[e^{kb}\right]_{b=R_1}^{R_2} + \sigma(4\pi R_3{}^2)}{4\pi r^2 \varepsilon_o}$$

$$\Rightarrow \qquad E = \frac{\left(\frac{C}{k}\right)\left[e^{kR_2} - e^{kR_1}\right] + \sigma(R_3{}^2)}{r^2 \varepsilon_o}.$$

Summarizing what we know, and remembering that we can substitute in C = 10^{-12} coulombs per meter, $\sigma = 10^{-12}$ coulombs per square meter, R_1 = 1 meter, R_2 = 2 meters, and R_3 = 3 meters, we have:

--For $r < R_1$: $E = 0.$

--For $R_1 < r < R_2$:

$$E = \frac{\left(\frac{C}{k}\right)\left[e^{kr} - e^{kR_1}\right]}{r^2 \varepsilon_o}$$

$$= \frac{\left(\frac{10^{-12}}{1}\right)\left[e^{(1)r} - e^{(1)(1)}\right]}{r^2\left(8.85x10^{-12}\right)}$$

$$= \frac{(.113)\left[e^{r} - 2.7\right]}{r^2} \qquad \text{where } 1 < r < 2.$$

--For $R_2 < r < R_3$: $E = 0.$

--For $R_3 < r$:

$$E = \frac{\left(\frac{C}{k}\right)\left[e^{kR_2} - e^{kR_1}\right] + \sigma R_3{}^2}{r^2 \varepsilon_o}$$

$$= \frac{\left(\frac{10^{-12}}{1}\right)\left[e^{(1)(2)} - e^{(1)(1)}\right] + (10^{-12})(3)^2}{r^2\left(8.85x10^{-12}\right)}$$

$$= \frac{1.545}{r^2} \qquad \text{where } r > 3.$$

The graph of these functions, each evaluated in its own region, is presented on the next page in Figure VI.

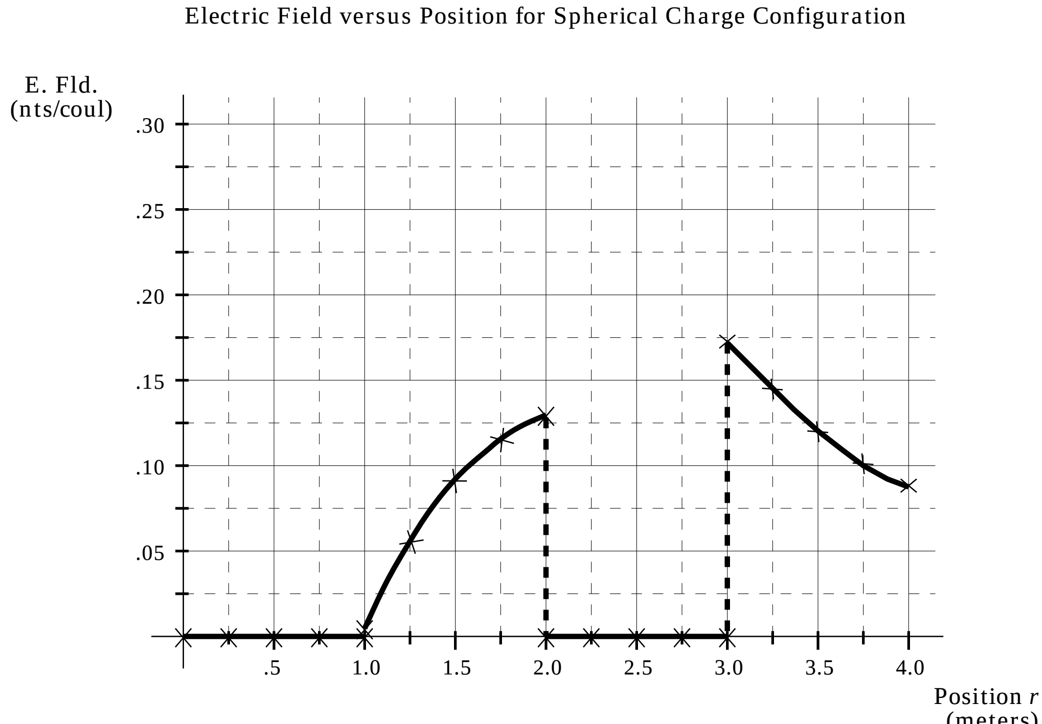


FIGURE VI

c.) Electric field functions are evidently not continuous functions, as the graph shows.

14.5) The system is shown in Figure VII to the right.

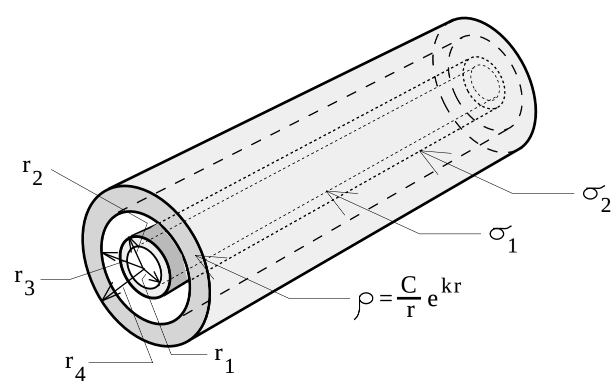


FIGURE VII

a.) The outside pipe must be an insulator as it has a volume charge density associated with it (there can be no free charge inside a conductor as all such charge will migrate as far away from like charge as possible, redistributing itself on a conductor's outside surface).

A Gaussian surface between r_1 and r_2 will have charge enclosed within it due to the surface charge density σ_1 on the inside pipe's surface. That means there will be an electric flux through the Gaussian surface which, in turn, means there will be an electric field in that region. A conductor cannot have an electric field within its boundary, hence the material making up the inside pipe must be an insulator.

b.) As was the case with the spherical-symmetry-problem of the same kind (Problem 14.4), we must derive electric field expressions for each region. Doing so yields:

--For $r < r_1$: A cylindrical Gaussian surface within this region will have no charge enclosed within it, hence the electric field in this region will be zero.

--For $r_1 < r < r_2$: Inside the innermost pipe, the charge enclosed within a cylindrical Gaussian surface of length L will be the consequence of the surface charge density σ_1. As such:

$$\int_S \mathbf{E} \bullet d\mathbf{S} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Rightarrow \quad \int_S E(dS)\cos 0^o = \frac{\sigma_1(2\pi r_1 L)}{\varepsilon_o}$$

$$\Rightarrow \quad E\int_S (dS) = \frac{\sigma_1(2\pi r_1 L)}{\varepsilon_o}$$

$$\Rightarrow \quad E(2\pi r L) = \frac{\sigma_1(2\pi r_1 L)}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\sigma_1 r_1}{r\varepsilon_o}.$$

--For $r_2 < r < r_3$: Outside the inner pipe, the charge enclosed within a cylindrical Gaussian surface of length L will be the consequence of the surface charge density on the inside and outside surfaces. As such:

$$\int_S \mathbf{E} \bullet d\mathbf{S} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Rightarrow \quad E(2\pi r L) = \frac{\sigma_1(2\pi r_1 L) - \sigma_2(2\pi r_2 L)}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\sigma_1 r_1 - \sigma_2 r_2}{r\varepsilon_o}.$$

--For $r_3 < r < r_4$: Inside the outer pipe, there is a volume charge density to consider. Remembering we must include ALL the charge enclosed within the Gaussian surface (i.e., including the charge on the inner pipe's surfaces), we can write:

$$\int_S \mathbf{E} \bullet d\mathbf{S} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Rightarrow \quad E(2\pi rL) = \frac{\int_{a=r_3}^{r} \rho dV + \sigma_1(2\pi r_1 L) - \sigma_2(2\pi r_2 L)}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\int_{a=r_3}^{r} \left[(C/a)e^{ka}\right]\left[(2\pi aL)da\right] + \sigma_1(2\pi r_1 L) - \sigma_2(2\pi r_2 L)}{(2\pi rL)\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{C\int_{a=r_3}^{r} (e^{ka})da + \sigma_1(r_1) - \sigma_2(r_2)}{(r)\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\left[(C/k)e^{ka}\right]_{a=r_3}^{r} + \sigma_1(r_1) - \sigma_2(r_2)}{(r)\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{(C/k)\left[e^{kr} - e^{kr_3}\right] + \sigma_1(r_1) - \sigma_2(r_2)}{(r)\varepsilon_o}.$$

--For $r > r_4$: Outside the outer pipe, ALL the charge in the entire system is enclosed within a Gaussian surface. As such, we can write:

$$\int_S \mathbf{E} \bullet d\mathbf{S} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Rightarrow \quad E(2\pi rL) = \frac{\int_{a=r_3}^{r_4} \rho dV + \sigma_1(2\pi r_1 L) - \sigma_2(2\pi r_2 L)}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\int_{a=r_3}^{r_4} \left[(C/a)e^{ka}\right]\left[(2\pi aL)da\right] + \sigma_1(2\pi r_1 L) - \sigma_2(2\pi r_2 L)}{(2\pi rL)\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\left[(C/k)e^{ka}\right]_{a=r_3}^{r_4} + \sigma_1(r_1) - \sigma_2(r_2)}{(r)\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{(C/k)\left[e^{kr_4} - e^{kr_3}\right] + \sigma_1(r_1) - \sigma_2(r_2)}{(r)\varepsilon_o}.$$

Summarizing what we know and putting in C = 10^{-12} coulombs per square meter, σ_1 = 10^{-12} coulombs per square meter, σ_2 = $2\text{x}10^{-12}$ coulombs per square meter (this is the charge's magnitude--the negative sign has already been incorporated into the problem via Gauss's Law), r_1 = 1 meter, r_2 = 2 meters, r_3 = 3 meters, and r_4 = 4 meters, we get:

--For $r < r_1$: $E = 0.$

--For $r_1 < r < r_2$:

$$E = \frac{\sigma_1 r_1}{r\varepsilon_o}$$

$$= \frac{(10^{-12})(1)}{(8.85\text{x}10^{-12})r}$$

$$= \frac{.113}{r} \qquad \text{where } 1 < r < 2.$$

--For $r_2 < r < r_3$:

$$E = \frac{\sigma_1 r_1 - \sigma_2 r_2}{r\varepsilon_o}$$

$$= \frac{(10^{-12})(1) - (2\text{x}10^{-12})(2)}{(8.85\text{x}10^{-12})r}$$

$$= -\frac{.339}{r} \qquad \text{where } 2 < r < 3.$$

--For $r_3 < r < r_4$:

$$E = \frac{\left[\frac{C}{k}\left[e^{kr} - e^{kr_3}\right]\right] + \sigma_1 r_1 - \sigma_2 r_2}{r\varepsilon_o}$$

$$= \frac{\left[\left(\frac{10^{-12}}{1}\right)\left[e^{(1)r} - e^{(1)(3)}\right]\right] + (10^{-12})(1) - (2\text{x}10^{-12})(2)}{(8.85\text{x}10^{-12})r}$$

$$= \frac{.113e^{r} - 2.6}{r} \qquad \text{where } 3 < r < 4.$$

--For $r_4 < r$:

$$E = \frac{\left[\frac{C}{k}\left[e^{kr_4} - e^{kr_3}\right]\right] + \sigma_1 r_1 - \sigma_2 r_2}{r\varepsilon_o}$$

$$= \frac{\left[\left(\frac{10^{-12}}{1}\right)\left[e^{(1)(4)} - e^{(1)(3)}\right]\right] + (10^{-12})(1) - (2x10^{-12})(2)}{(8.85x10^{-12})r}$$

$$= \frac{3.56}{r} \qquad \text{where } 4 < r.$$

Putting it all together in Figure VIII, we get:

Electric Field versus Position for Cylindrical Charge Configuration

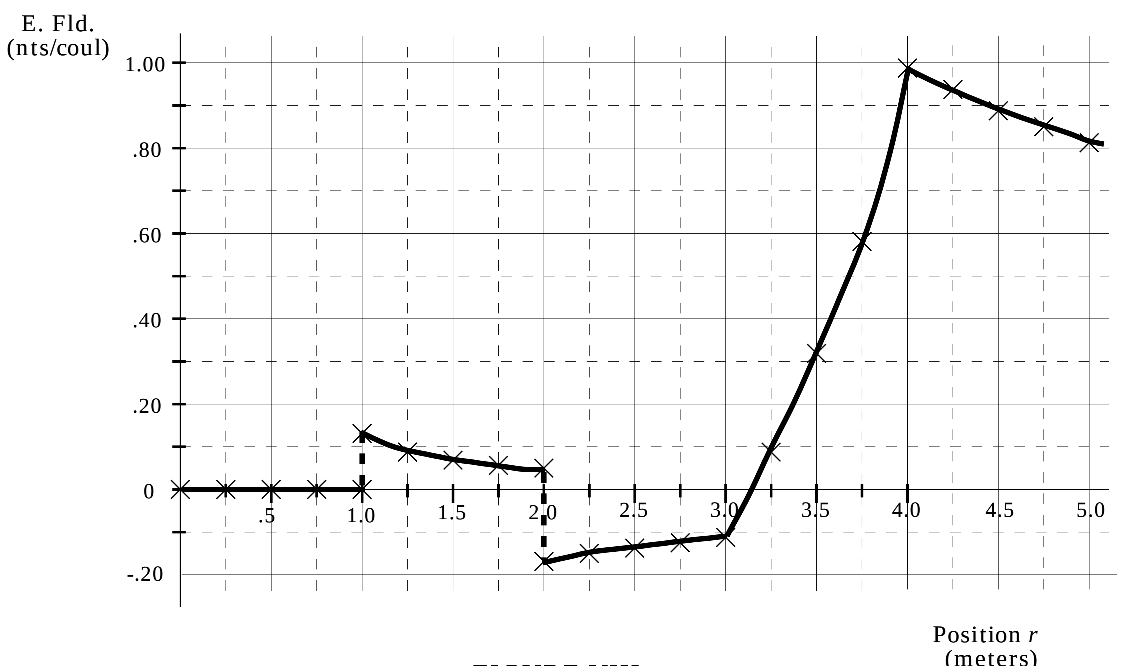


FIGURE VIII

14.6) The easiest possibility is a volume charge density function that is constant. Assuming that is the case, Gauss's Law yields:

$$\int_S \mathbf{E} \bullet d\mathbf{S} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Rightarrow \quad \int_s (E)dS(\cos 0^o) = \frac{\rho V}{\varepsilon_o}$$

$$\Rightarrow \quad \left[\frac{kR^6}{6\varepsilon_o r}\right]\int_s dS = \frac{\rho\left[\pi R^2 L\right]}{\varepsilon_o}$$

$$\Rightarrow \quad \left[\frac{kR^6}{6\varepsilon_o r}\right](2\pi r L) = \frac{\rho\left[\pi R^2 L\right]}{\varepsilon_o}$$

$$\Rightarrow \quad \rho = \frac{kR^4}{3}.$$

A second possibility is to assume that the volume charge density function varies with the distance from the central axis. In that case, we can use Gauss's Law in conjunction with a cylindrical Gaussian surface to determine the electric field outside the rod, write out the q_{encl} expression in terms of the volume charge density function ρ, and then determine ρ by comparing our integral expression with the known electric field expression. Doing so yields:

$$\int_S \mathbf{E} \bullet d\mathbf{S} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Rightarrow \quad \int_s (E)dS(\cos 0^o) = \frac{\int_{a=0}^{R} \rho dV}{\varepsilon_o}$$

$$\Rightarrow \quad E\int_s dS = \frac{\int_{a=0}^{R} \rho\left[\left((2\pi a)da\right)L\right]}{\varepsilon_o}$$

$$\Rightarrow \quad \left[\frac{kR^6}{6\varepsilon_o r}\right](2\pi r L) = \frac{2\pi L\int_{a=0}^{R} \rho\left[(a)da\right]}{\varepsilon_o}$$

$$\Rightarrow \quad \left[\frac{kR^6}{6}\right] = \int_{a=0}^{R} \rho\left[(a)da\right].$$

By inspection, $\rho = ka^4$.

Not clear? Consider:

--If $\rho = k$, the integral would equal $ka^2/2$. After evaluating between a = 0 and a = R, the right-hand side of Gauss's Law would equal $kR^2/2$.

--If $\rho = ka$, the integral would equal $ka^3/3$. After evaluating between a = 0 and a = R, the right-hand side of Gauss's Law would equal $E = kR^3/3$.

--If $\rho = ka^n$, the integral would equal $ka^{(n+2)}/(n+2)$. After evaluating between a = 0 and a = R, the right-hand side of Gauss's Law would equal $E = kR^{(n+2)}/(n+2)$.

--Noting this, the right-hand side of Gauss's Law will equal the left-hand side (i.e., $kR^6/6$) if the volume charge density function is ka^4.

14.7)

a.) Assuming the disk's thickness is t, the volume charge density for the disk is:

$$\begin{aligned}\rho &= Q/V \\ &= Q/[\pi R^2 t] \\ &= Q/[\pi(1\text{ m})^2(.02\text{ m})] \\ &= 15.9Q.\end{aligned}$$

Using a cylindrical Gaussian plug that extends through the disk <u>an equal distance on either side</u> (see Part I of this chapter if this is not clear), we get a near-field derivation that yields:

$$\int_S \mathbf{E} \bullet d\mathbf{S} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Rightarrow \quad EA + EA = \frac{\int \rho dV}{\varepsilon_o}$$

$$\Rightarrow \quad 2EA = \frac{15.9Q\int dV}{\varepsilon_o}$$

$$\Rightarrow \quad 2EA = \frac{15.9Q(At)}{(8.85x10^{-12})}$$

$$\Rightarrow \quad 2E = \frac{15.9Q(.02)}{(8.85x10^{-12})}$$

$$\Rightarrow \quad E = 1.8x10^{10}Q.$$

b.) A surface charge density function for this configuration must allow us to determine the amount of charge underneath, so to speak, an area-section on the disk's surface.

As the total charge within the disk is Q while the total surface area is $\pi R^2 = \pi(1\text{ m})^2 = 3.14\text{ m}^2$, we can write:

$$\begin{aligned}\sigma &= Q/A \\ &= Q/3.14 \\ &= .318Q.\end{aligned}$$

c.) Treating the disk like a sheet of charge and using a cylindrical Gaussian plug that extends through the sheet of charge an equal distance on either side, we get:

$$\int_S \mathbf{E} \bullet d\mathbf{S} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Rightarrow \quad EA + EA = \frac{\int \sigma dA}{\varepsilon_o}$$

$$\Rightarrow \quad 2EA = \frac{.318Q\int dA}{\varepsilon_o}$$

$$\Rightarrow \quad 2EA = \frac{.318QA}{(8.85x10^{-12})}$$

$$\Rightarrow \quad E = 1.8x10^{10}Q.$$

This is the same near-field expression we calculated using the volume charge density function in Part a. It is interesting to note that this is also numerically equal to $\sigma/2\varepsilon_o$.

d.) Consider a circular hoop of radius r with total charge dq distributed uniformly on its surface. Define a differential charge dq' at some arbitrary position on the hoop. That differential charge will produce a differential electric field dE' at x (see Figure IX).

When we integrate to get the net field dE due to all the charge dq in the hoop, the y components will add to zero due to the symmetry of the configu-

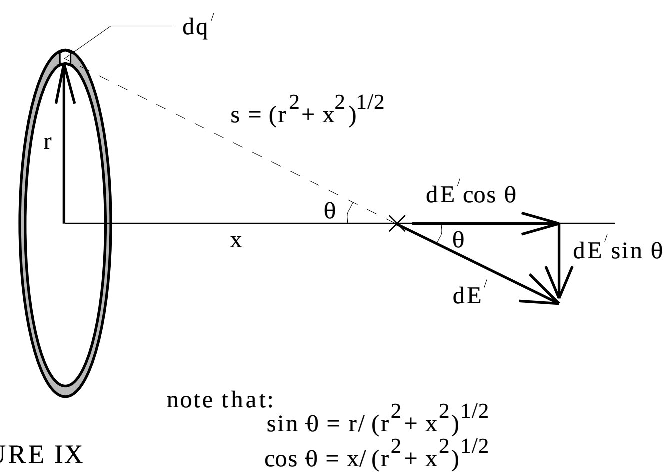


FIGURE IX

ration. That means we can forget them and focus solely on the x components of the field. Put another way, the net field dE from the hoop will equal the x component of the vector sum of all the dE' quantities, or:

$$dE = \int dE'(\cos\theta).$$

Doing that integral yields:

$$\begin{aligned} dE &= \int dE'_x \\ &= \int \left[\frac{1}{4\pi\varepsilon_o}\frac{dq'}{s^2}\right]\cos\theta \\ &= \int \left[\frac{1}{4\pi\varepsilon_o}\frac{dq'}{\left((r^2+x^2)^{1/2}\right)^2}\right]\left(\frac{x}{(r^2+x^2)^{1/2}}\right) \\ &= \left[\frac{x}{4\pi\varepsilon_o\left(r^2+x^2\right)^{3/2}}\int dq'\right] \\ &= \frac{(x)dq}{4\pi\varepsilon_o\left(r^2+x^2\right)^{3/2}}. \end{aligned}$$

Again, the differential charge dq in this expression is the total charge on the hoop.

Having an expression for the magnitude and direction of the electric field of an arbitrary hoop along its central axis, we can now deal with our disk (see Figure X). Noting that $dq = \sigma dA$, where $dA = (2\pi r)dr$ and $\sigma = (Q/\pi R^2)$, we can write:

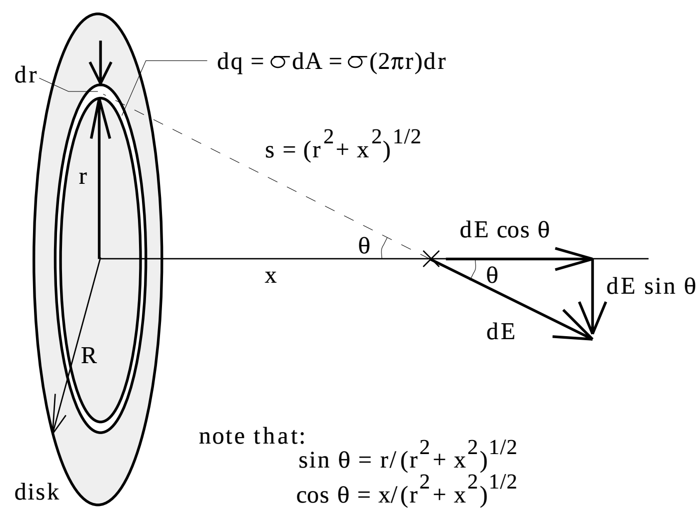


FIGURE X

$$E_{actual} = \int dE$$

$$= \int \frac{(x)dq}{4\pi\varepsilon_o (r^2 + x^2)^{3/2}}$$

$$= \frac{x}{4\pi\varepsilon_o} \int \frac{\sigma dA}{(r^2 + x^2)^{3/2}}$$

$$= \frac{x}{4\pi\varepsilon_o} \int \frac{\sigma[(2\pi r)dr]}{(r^2 + x^2)^{3/2}}$$

$$= \frac{x\sigma}{2\varepsilon_o} \int_{r=0}^{R} \frac{r}{(r^2 + x^2)^{3/2}} dr$$

$$\Rightarrow \quad E_{actual} = \frac{\sigma x}{2\varepsilon_o} \left[\frac{1}{x} - \frac{1}{(R^2 + x^2)^{1/2}} \right].$$

e.) As we get away from the disk, the field strength will decrease. That means that the constant $\sigma/2\varepsilon_o$ value we derived in the chapter for the near-field situation will become further and further off as one proceeds out along the x axis. We want to know at what x that near-field value is 95% of the actual value. Using the actual field expression derived in Part d and the near-field information derived in Parts a and/or b we can write:

$$.95E_{near-fld} = \frac{\sigma x}{2\varepsilon_o} \left[\frac{1}{x} - \frac{1}{\left(R^2 + x^2\right)^{1/2}} \right]$$

$$\Rightarrow \quad .95\left[1.8x10^{10}Q\right] = \frac{\left(\frac{Q}{\pi R^2}\right)x}{2\varepsilon_o} \left[\frac{1}{x} - \frac{1}{\left(R^2 + x^2\right)^{1/2}} \right]$$

$$\Rightarrow \quad .95(1.8x10^{10})\left[2(8.85x10^{-12})\right]\pi(1)^2 = \left[1 - \frac{x}{\left(R^2 + x^2\right)^{1/2}} \right]$$

$$\Rightarrow \quad .95 = \left[1 - \frac{x}{\left(R^2 + x^2\right)^{1/2}} \right]$$

$$\Rightarrow \quad x = .05R.$$

Evidently, the Gaussian electric field is good to 5% at .05R meters from the surface. For our case in which R = 1 meter, the near-point Gaussian expression is within 5% of the actual electric field value when at 5 centimeters from the disk's surface.